The 11 most frightening words in education

Robert Wilne
Director for Secondary
NCETM
5 November 2015

National Centre for Excellence in the Teaching of Mathematics

Holmes or Watson?

National Centre for Excellence in the Teaching of Mathematics

Which calculation is the **odd one out**? Explain your reasoning.

- 753×1.8
- $75.3 \times 20 - 75.3 \times 2$
- $753 + 753 \div 5 \times 4$
- $750 \times 1.8 + 30 \times 0.18$
- $(75.3 \times 3) \times 6$
- 7.53×1800

Premises

National Centre for Excellence in the Teaching of Mathematics

I do believe that

- some pupils do better in maths assessments than others
- some pupils "get" maths more quickly than others
- some pupils have the cognitive architecture that means they see patterns and structures more quickly, and remember them more readily, than others do.
- But I don't **know** who
- And I can't **predict** who
- And pupils' brains **change**

ACME, *Raising the bar* (2012)

National Centre for Excellence in the Teaching of Mathematics

- Potential heavy users of mathematics should experience a **deep, rich, rigorous and challenging** mathematics education, rather than being accelerated through the school curriculum.
- Accountability measures should allow, support and reward an **approach focused on depth of learning**, rather than rewarding early progression to the next Key Stage.
- **Investment in a substantial fraction of 5-16 year olds** with the potential to excel in mathematics, rather than focussing attention on the top 1% (or so), is needed to increase the number of 16+ students choosing to study mathematics-based subjects or careers.

Aims and ethos of the UK curriculum

All pupils must

become **FLUENT** in the
...s of mathematics,

SOLVE PROBLEMS by
their mathematic
routine an
problem
sophisticatio
down prob
simpler step
seekin

REASON MATHEMATICALLY
by following a line of enquiry,
conjecturing relationships
and generalisations, and
developing an argument,
justification or proof using
mathematical language.

Note that

All pupils

1. become **FLUENT** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
2. **REASON MATHEMATICALLY** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language.
3. can **SOLVE PROBLEMS** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

The vision of the new curriculum

National Centre
for Excellence in the
Teaching of Mathematics

All pupils become **fluent** in the fundamentals of mathematics, including through **varied and frequent practice** with increasingly complex problems over time, **so that pupils develop conceptual understanding** and the ability to **recall and apply knowledge** rapidly and accurately; **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language; **solve** problems by applying their mathematics to a variety of **routine and non-routine problems** with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions. The expectation is that the **majority of pupils** will move through the programmes of study **at broadly the same pace**. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered **rich and sophisticated problems before any acceleration** through new content. Those who are not sufficiently fluent should consolidate their understanding, including through **additional practice, before moving on**.

The vision of the new curriculum

National Centre
for Excellence in the
Teaching of Mathematics

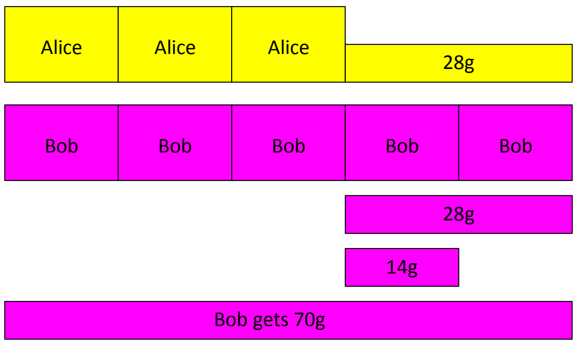
All pupils become **fluent** in the fundamentals of mathematics, including through **varied and**
frequent practice with increasingly complex problems over time, **so that pupils**
develop conceptual understanding and the ability to **recall and apply**
knowledge rapidly and accurately; **...hematically** by following a line of enquiry,
conjecturing relationships and generalisati... ...tification or proof using mathematical
language; **solve** problems by appl... **...utine and non-routine**
problems with increasing sophisticatio... ...wn problems into a series of simpler steps and

intelligent practice

persevering in seeking solutions. The expectation is that the **majority of pupils** will move through the programmes of study **at broadly the same pace**. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered **rich and sophisticated problems before any acceleration** through new content. Those who are not sufficiently fluent should consolidate their understanding, including through **additional practice, before moving on**.

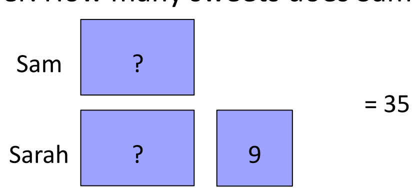
How would your pupils tackle this?

- I share some sultanas between Alice and Bob in the ratio 3:5. Alice gets 28g fewer sultanas than Bob. How many grams of sultanas does Bob get?

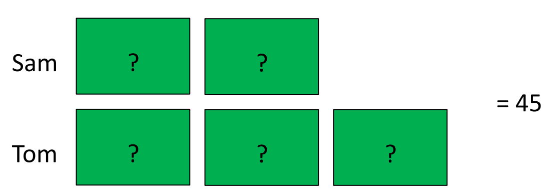


Box / Bar representations

1. Sam has 9 fewer sweets than Sarah. They have 35 sweets altogether. How many sweets does Sam have?



2. Sam and Tom share 45 marbles in the ratio 2:3. How many more marbles does Tom have than Sam?

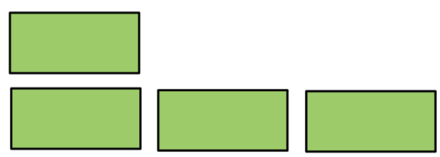


Box / Bar going deeper

3. Emily makes 250 grams of salad. 15% of the weight is tomato, 25% is cucumber, and the rest is lettuce. How many grams of lettuce does she use?

4. A triangle has been drawn carefully. The biggest angle is 20° larger than the second biggest angle, and is 40° larger than the smallest angle. Work out how big each angle is.

5. **Make up** a word puzzle that you could solve with this diagram:



= 60

Box / Bar deeper still

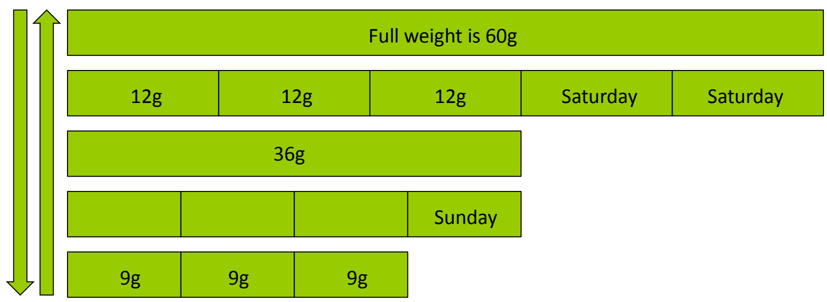
National Centre
for Excellence in the
Teaching of Mathematics

In Year 4, 45% of the pupils are boys. There are 14 fewer boys than girls in Year 4. How many girls are there in Year 4?

If you use different values other than 14, will the answers you get to the question "How many girls are there in Year 4?" **always, sometimes or never** make sense?

Box / Bar challenge

- On Saturday I opened a new packet of sultanas and gave Charlie 40% of them. On Sunday I gave him 25% of the remaining sultanas. Today I have given him the remainder of the sultanas, which weigh 27g. What was the weight of the packet when it was full?



Box / Bar models are good because …

- can be explored valuably by **ALL** pupils, irrespective of prior attainment
- fit naturally these sorts of questions
- enable **ALL** pupils to calculate answers accurately and efficiently
- help **ALL** pupils focus on the mathematics in the problems
- can be developed over time from simple to more complex problems
- enable and support **ALL** pupils to think and reason about problems of increasing complexity, and do so with increasing confidence and sophistication.

Is this increasing abstraction?

When I DRAW 3 boxes showing
Alice's sultanas and 5 boxes
showing Bob's, then
these differ by
the sa

I COULD draw 3 boxes
representing Alice's sultanas and
5 boxes representing Bob's, and
then I WOULD see that these
WOULD differ by
WOULD re

$28 \div (5 - 3) = 14$

$14 \times 5 = 70$

Finished. Please may I have some rich and sophisticated problems to do now?

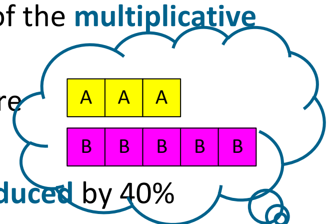
I share some sultanas between Alice and Bob in the ratio 3:5. Alice gets 28g fewer sultanas than Bob. How many grams of sultanas does Bob get?

How would your pupils respond?

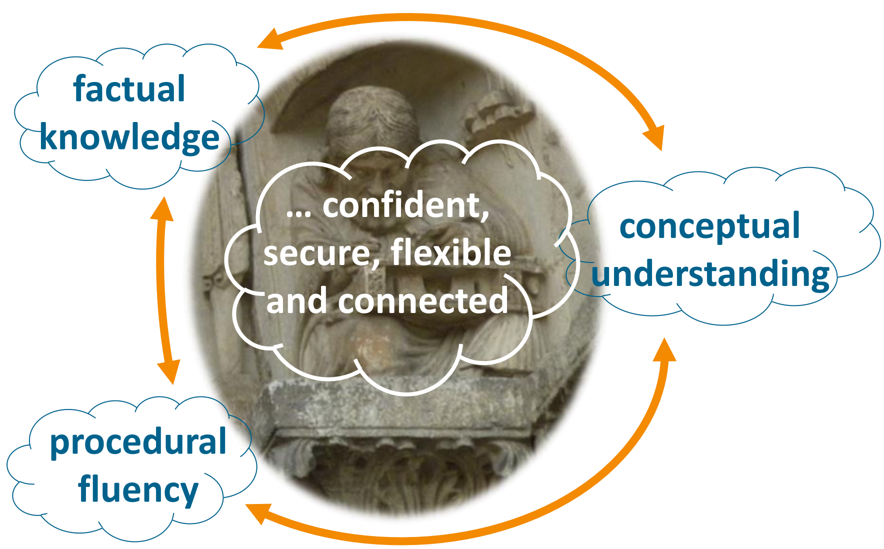
- I share some sultanas between Alice and Bob in the ratio 3:5. Alice gets 28g fewer sultanas than Bob. How many grams of sultanas does Bob get?
- I share some sultanas between Alice and Bob in the ratio 6:10. Alice gets 28g fewer sultanas than Bob. How many grams of sultanas does Bob get?
- I share some sultanas between Alice and Bob, so that Bob gets $\frac{5}{8}$ of all the sultanas. Alice gets 28g fewer sultanas than Bob. How many grams of sultanas does Bob get?
- I share some sultanas between Alice and Bob, so that Alice gets 60% of what Bob gets. Alice gets 28g fewer sultanas than Bob. How many grams of sultanas does Bob get?

Even good models have limitations

- Drawing boxes / bars focuses pupils' attention on the **additive** structure of the Alice and Bob problem: Alice has 2 boxes / bars **fewer** than Bob, Bob has 2 boxes / bars **more**.
- The risk is that we don't ensure that pupils also acquire deep conceptual understanding of the **multiplicative** structure of the problem: that
  - Alice's share is ⅗ **of** Bob's share
  - Bob's share is ⅝ **of** the total
  - Alice's share is Bob's share **reduced** by 40%
  - the **scale factor** from Alice's share to Bob's is 1⅔.



ALL pupils should be expected to develop …



Knowledge

National Centre
for Excellence in the
Teaching of Mathematics

Pupils know

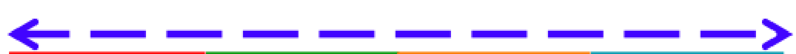
- **THAT** … 7 × 8 = 56. They have **FACTUAL** knowledge.
- **HOW** … te 37 × 28.
- **WHY** … e
  DING.
- **TO** … mate the
  speed of a rocket t… es 550m in 7.1s
  They can **APPLY** what they know to a **NON-ROUTINE PROBLEM**.

> *Pieces of information like 7 × 8 = 56 are not isolated facts. They are parts of the landscape, the territory of numbers, and that person knows them best who sees most clearly how they fit into the landscape and all the other parts of it.* John Holt

Conceptual understanding …

National Centre
for Excellence in the
Teaching of Mathematics

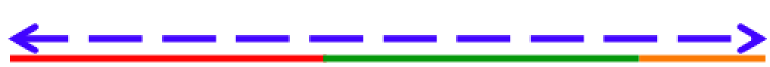
- 12 ÷ 3 = 4 …



- … because **four sticks of length 3** fit into a **gap of length 12**.

- So 12 ÷ 2.4 = 5, because …

- And 12 ÷ 5 = 2⅖, because …



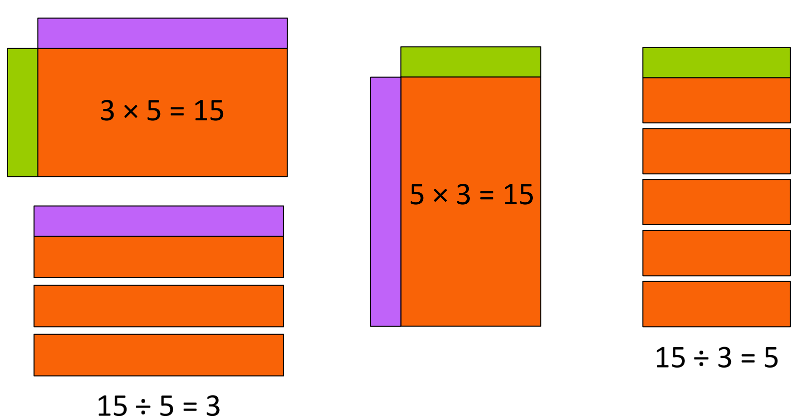
… leads pupils to generalisation

- $12 \div 3 = 4$ (4 sticks of length 3 fill a gap of length 12)
- so $12 \div 1.5 =$

- and $12 \div 0.3 =$
- and $120 \div 0.03 =$
- and $24 \div 6 =$
- and $12x \div 3x =$
- and $12a \div 3/b =$

Conceptual understanding …

3, 5, 15: three numbers four sentences



… enables procedural fluency …

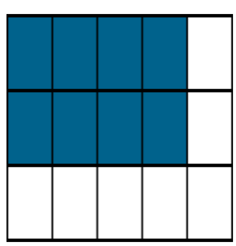
- 19×21

×	10	9	
20	200	180	
1	10	9	
			399

×	20	-1	
20	400	-20	
1	20	-1	
			399

… and facilitates generalisation

- $\frac{4}{5} \times \frac{2}{3}$



- $2\frac{2}{3} \times 3\frac{1}{2}$

×		
3		
$\frac{1}{2}$		
		$9\frac{1}{3}$

Pupils must be supported to develop the **standard algorithm** from this

… and facilitates generalisation

- $(2x-3)(3x+1)$

×	$2x$	-3
$3x$	$6x^2$	$-9x$
1	$2x$	-3

$\boxed{6x^2 - 7x - 3}$

- $6x^2 - 17x - 3$

×	x	-3
$6x$	$6x^2$	$-18x$
1	x	-3

$\boxed{6x^2 - 17x - 3}$

- **$12x^2 + 7x - 12$**
- **$x^2 + 6x + 13$**

"What's the same, what's different?"

$x^2 - 5x + 6$

$4x^2 - x - 14$

$x^2 + 5x + 6$

$4x^2 - 24x + 9$

$x^2 - 4x + 18$

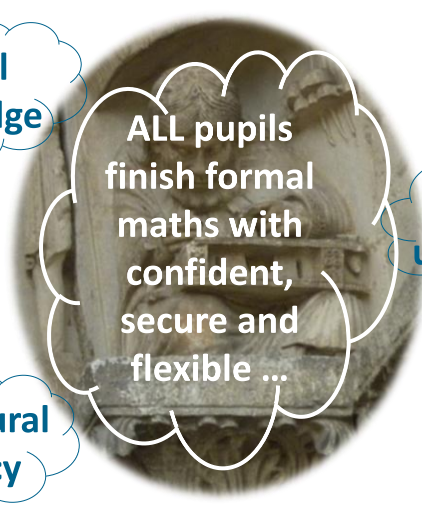
“Good” models / representations …

National Centre for Excellence in the Teaching of Mathematics

- can at first be explored “hands on” by **ALL** pupils irrespective of prior attainment
- arise naturally in the given scenario, so that they are salient and hence “sticky”
- can be implemented efficiently, and increase **ALL** pupils’ procedural fluency
- expose, and focus **ALL** pupils’ attention on, the underlying mathematics
- are extensible, flexible, adaptable and long-lived, from simple to more complex problems
- encourage, enable and support **ALL** pupils’ thinking and reasoning **about the concrete** to develop into thinking and reasoning **with increasing abstraction**.

“Mastery” is the outcome for ALL pupils

National Centre for Excellence in the Teaching of Mathematics



Teaching for mastery

National Centre
for Excellence in the
Teaching of Mathematics

If **pupils'** thinking and reasoning **about the concrete** is going to develop into thinking and reasoning **with increasing abstraction**, they need **us** their **teachers** to help make this happen. Crucial to this happening successfully are

- the **choice of the representation / model** with which we introduce a (new) concept
- the **reasoning** we cultivate and sharpen through the **discussions** we foster and steer
- the **misconceptions** we predict and confront as part of the sequence of **questions** we plan and ask
- the **conceptual understanding** we embed and deepen through the **intelligent practice** we design and prepare for the pupils to engage in and with.

Easy content, deep questioning

National Centre
for Excellence in the
Teaching of Mathematics

$$\frac{1}{3}+\frac{1}{6}=\frac{1}{2}$$

$$\frac{1}{5}+\frac{1}{?}=\frac{1}{?}$$

$$\frac{1}{?}+\frac{1}{?}=\frac{1}{5}$$

A Shanghai takeaway

National Centre
for Excellence in the
Teaching of Mathematics

A Shanghai takeaway

National Centre
for Excellence in the
Teaching of Mathematics

The eleven most frightening words in education are …

National Centre
for Excellence in the
Teaching of Mathematics

"I'm the G&T coordinator and you're going in a special group"

National Centre
for Excellence in the
Teaching of Mathematics

Agree or challenge?

Bouquets to: Dan Abramson, Michael Davies, Tony Gardiner, Debbie Morgan, Matt Nixon, Maths Hub Leads, Team NCETM, et al.

Brickbats to me!

National Centre
for Excellence in the
Teaching of Mathematics

robert.wilne@ncetm.org.uk
@NCETMsecondary

National Centre
for Excellence in the
Teaching of Mathematics